Vortex, wall-crossing and Seiberg-like duality in 3d

Chiung Hwang

Current Topics in String Theory: Conformal Field Theories
December 17, 2016, KIAS

work in progress with Piljin Yi & Yutaka Yoshida

Particle-vortex duality

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* Dualities of quantum field theories have been discussed in various contexts.
* Many dualities in 3d share a common feature: particle-vortex duality.
* An elementary field <-> a monopole operator
* Non-SUSY examples: a free Dirac fermion <-> QED3, bosonization, … Son '15, Wang-Senthil '15, Metlitski-Vishwanath '15, Aharony '15, Karch-Tong '16, Seiberg-Senthil-Wang-Witten '16, Murugan-Nastase '16, …
* SUSY examples: Seiberg-like duality, mirror symmetry, …
  Intriligator-Seiberg '96, Aharony '97, Giveon-Kutasov '08, …

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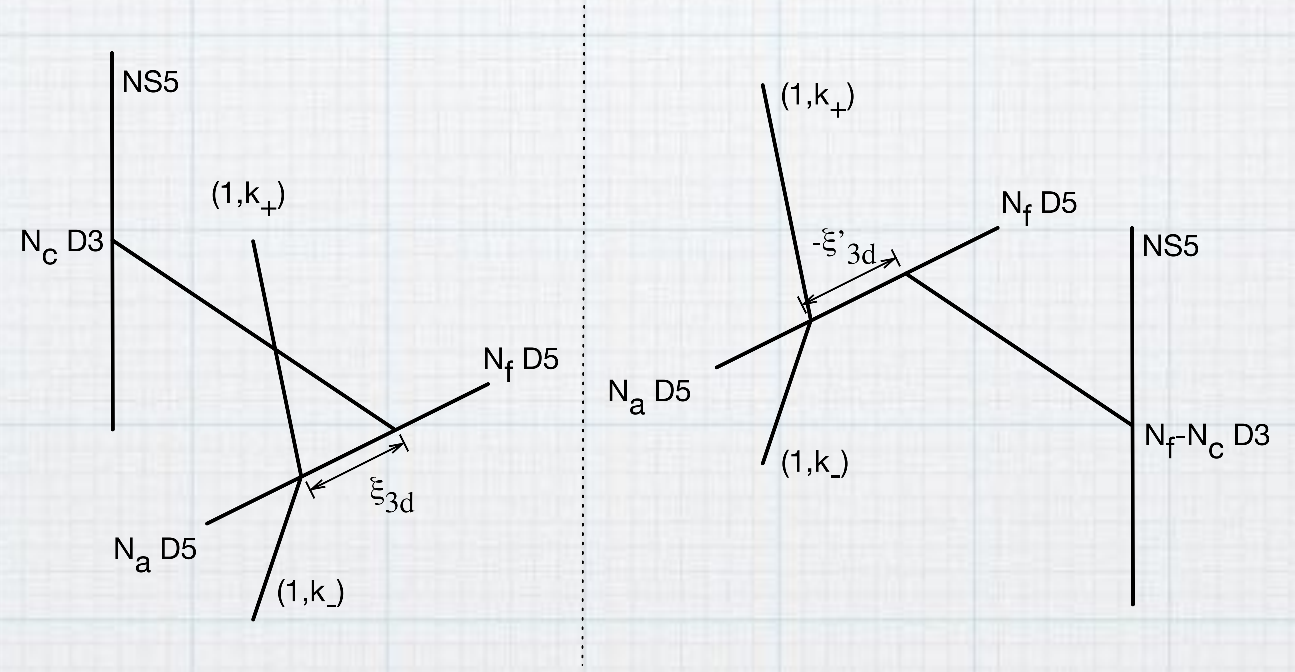
3d Seiberg-like duality

E.g., the Aharony duality Aharony '97, Benini-Closset-Cremonesi '11



$U(N_c)_k$ + (N_f,N_a) flavors ($|k| \leq (N_f-N_a)/2$)

<-> $U(N_f-N_c)_{-k}$ + (N_a,N_f) flavors + $N_f N_a$ mesons

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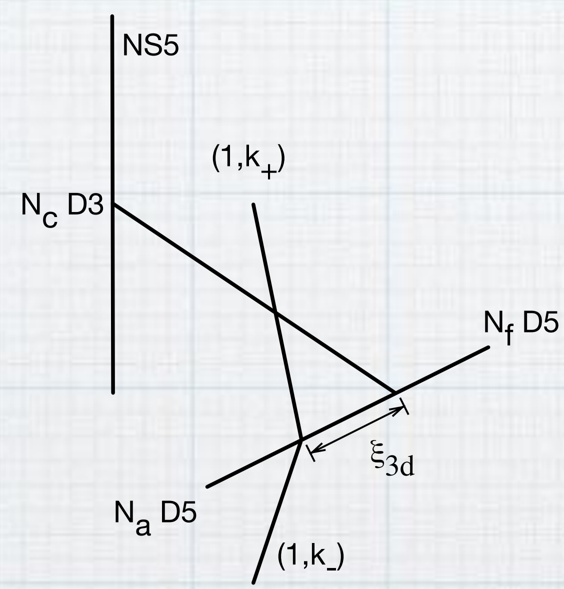

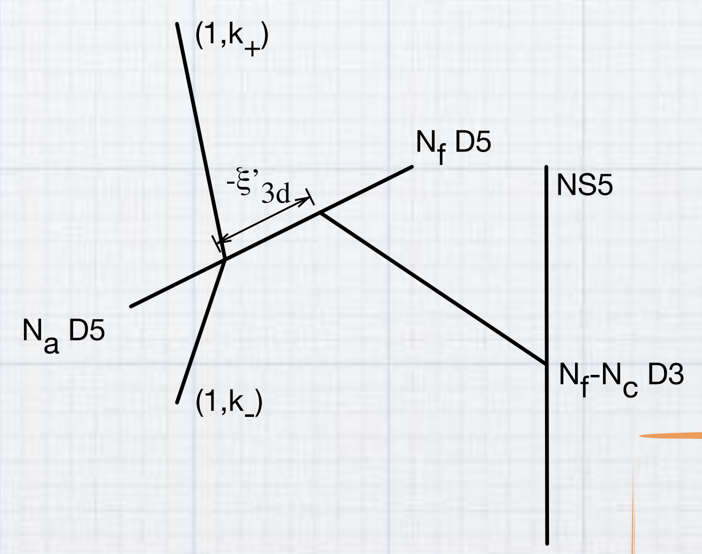

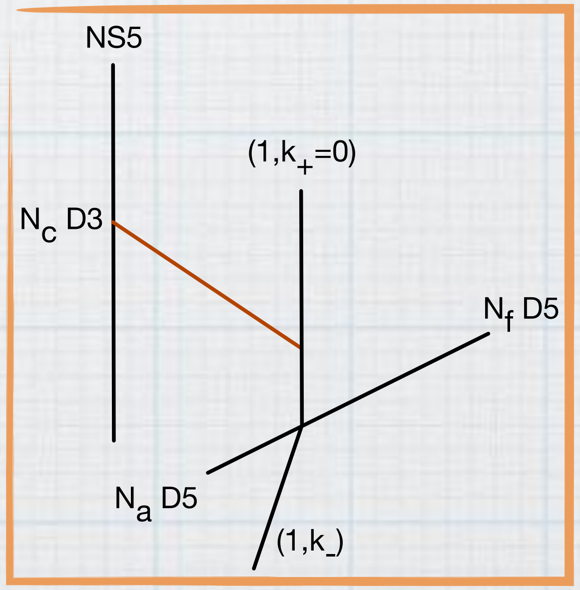

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<-> $U(N_f-N_c)_{-k}$ + (N_a,N_f) flavors + N_f N_a mesons + V_+,V_-

3d Seiberg-like duality

* Elementary fields V_+, V_- correspond to monopole operators in the original theory. -> the particle-vortex duality
* Behavior of vortices under the duality
* The vortex partition function = the partition function of the 3d theory on omega deformed $R_\Omega^2 \times S^1$
* Factorization of a 3d partition function Pasquetti '11, CH-H.Kim-J.Park '12, Taki '13... -> the Higgs branch localization of the partition function Fujitsuka-Honda-Yoshida '14, Benini-Peelaers '14
* A building block of a 3d partition function $Z = \sum Z_{\text{1-loop}} Z_{\text{vort}} Z_{\text{antiv}}$

Vortex quantum mechanics

* A different approach: the moduli space approximation of vortices Manton '82

* The sigma model description of the vortex moduli space

* The Witten index of vortex quantum mechanics is identified as the vortex partition function of the 3d theory.

Witten index of a 1d N=2 GLSM

The Witten index of a 1d N=2 GLSM can be computed by the supersymmetric localization. CH-J.Kim-S.Kim-J.Park '14, Cordova-Shao '14, Hori-H.Kim-P.Yi '14

$$I = \mathrm{Tr}\left[(-1)^F e^{-\beta H} e^{\sigma\cdot\mu}\right] = \frac{1}{|W|}\text{JK-Res}_{\vec{\eta}=\zeta\vec{1}}[g(u)d^r u]$$

$$g_{\text{vector}}(u) = \prod_{\alpha\in\Delta_G} 2\sinh\frac{-\alpha\cdot u}{2}$$

$$g_{\text{chiral}}(u) = \prod_{\rho\in R_\Phi}\prod_{\sigma\in F_\Phi}\frac{1}{2\sinh\frac{\rho\cdot u+\sigma\cdot\mu}{2}}$$

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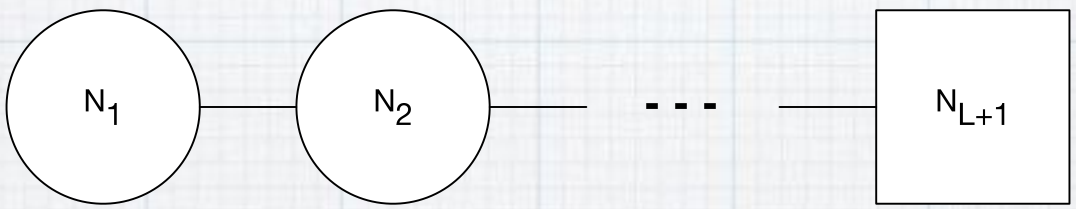
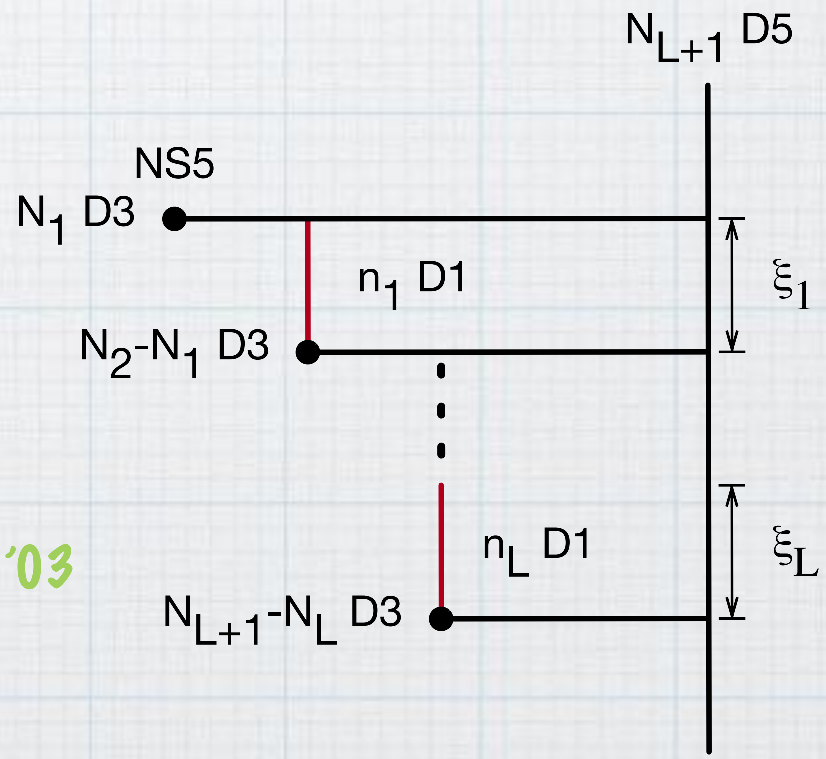
Examples

Example: $T_\rho[SU(N)]$

3d N=4 theories represented by the following quiver diagram:



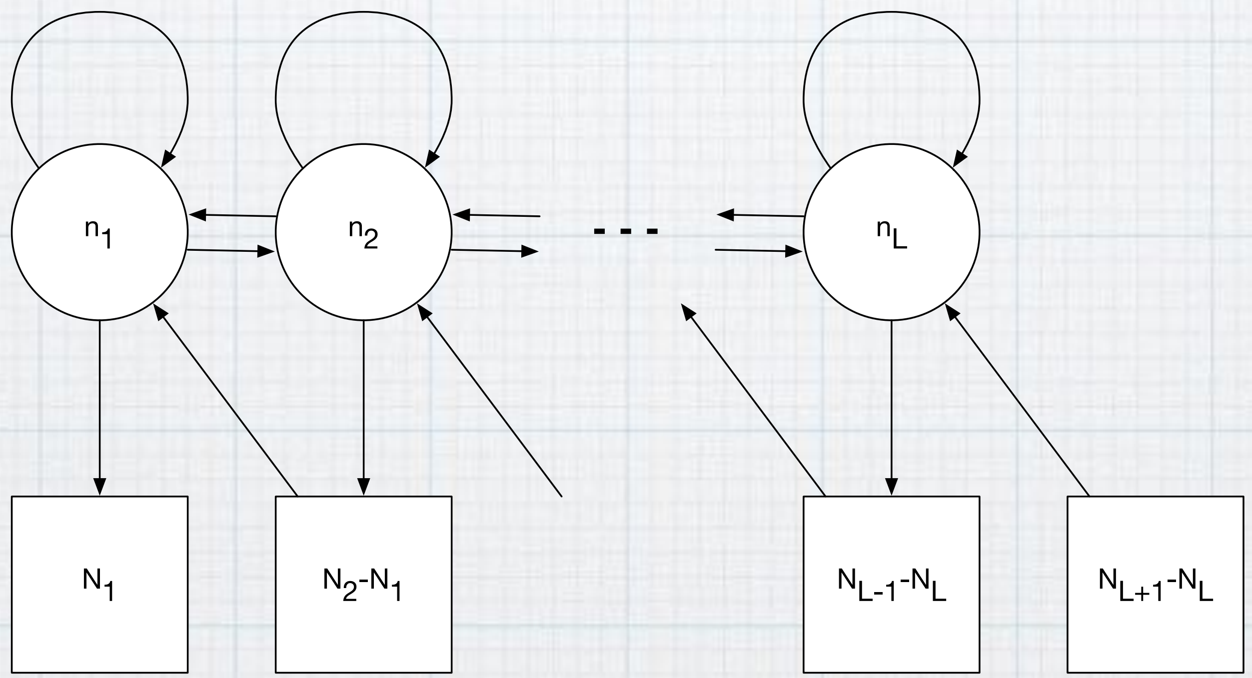
The type-IIB brane construction



D1s ending on D3s realize vortices. Hanany-Tong '03

Example: $T_\rho[SU(N)]$

The QM description Hanany-Tong '03, Bullimore-Dimofte-Gaiotto-Hilburn-H.Kim '16



with superpotential $$W = \sum_{i=2}^{L-1} J_i C_i I_{i+1} + \sum_{i=1}^{L-1} \mathrm{Tr} B_i C_i A_i - \sum_{i=1}^{L-1} \mathrm{Tr} B_{i+1} A_i C_i$$

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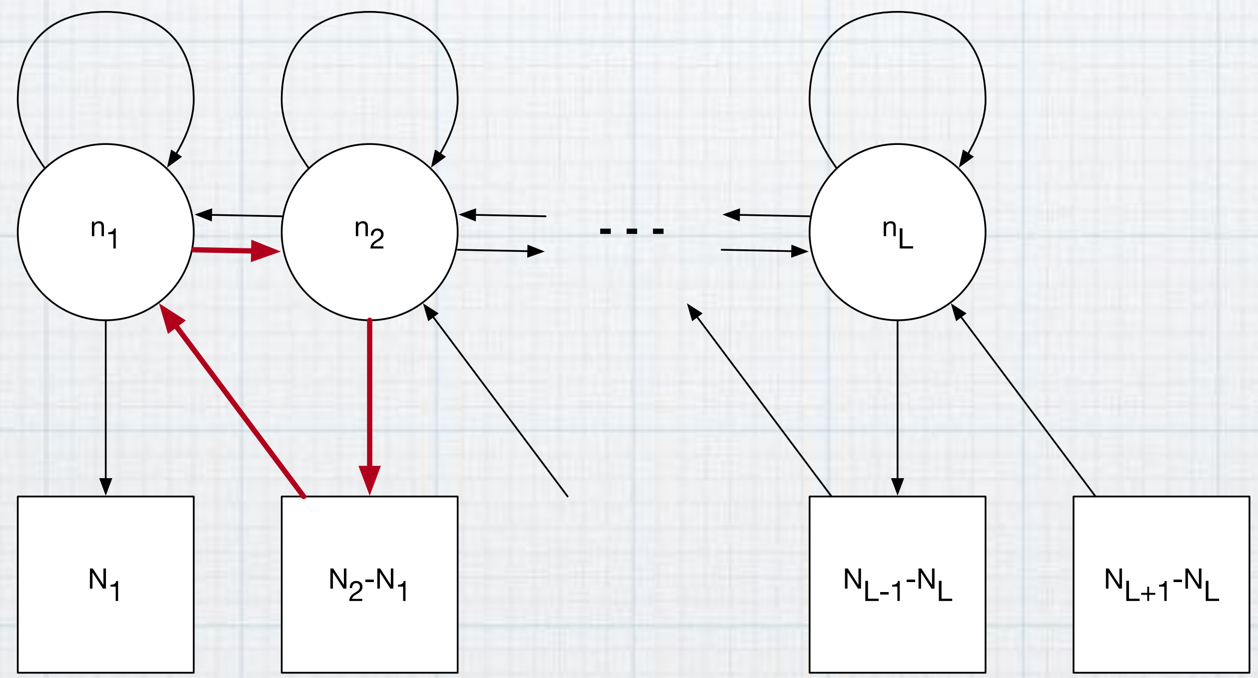

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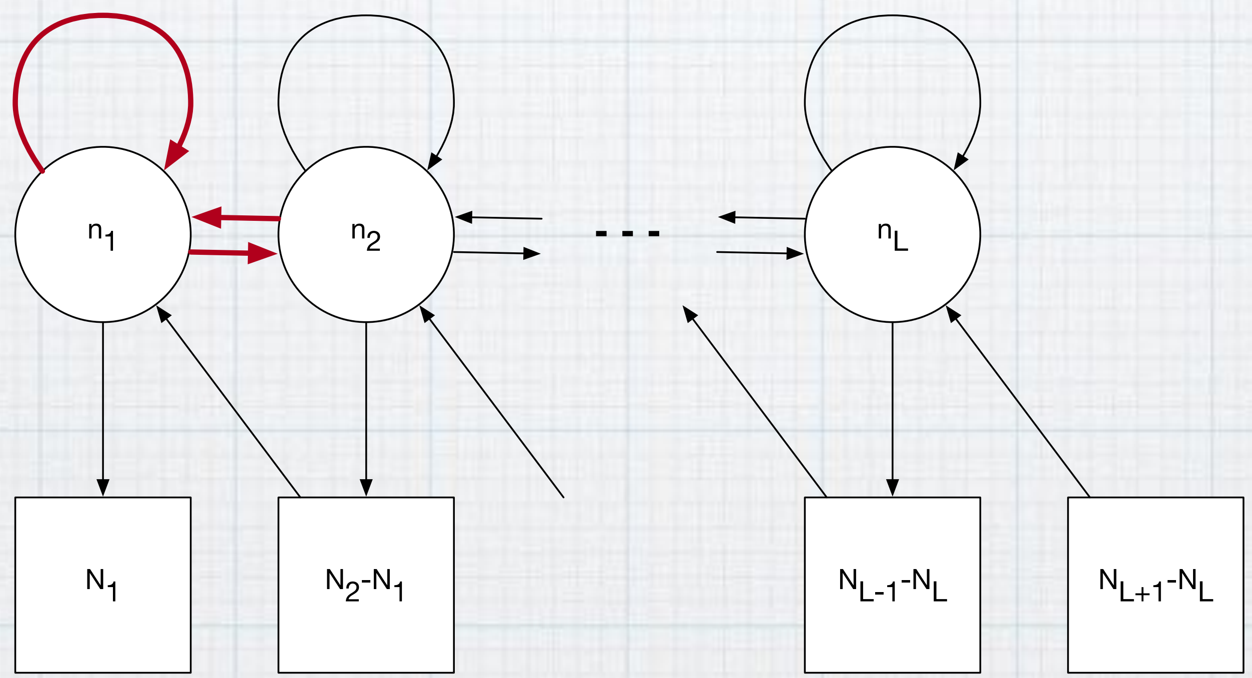

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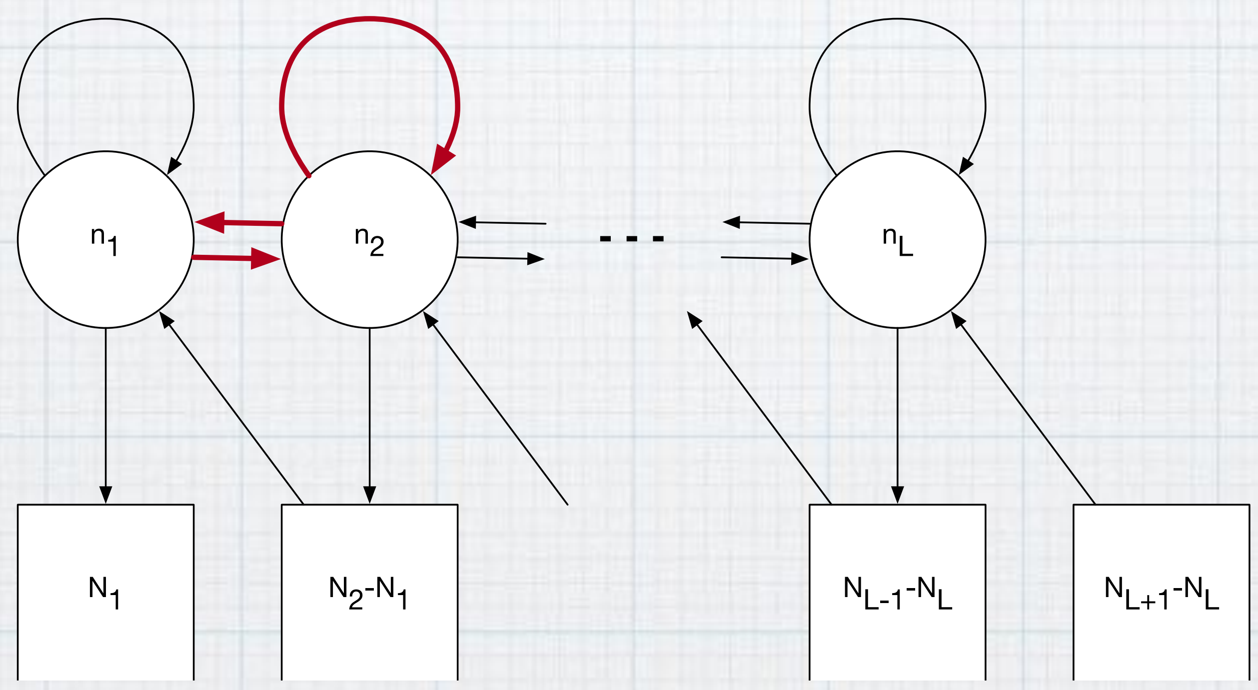

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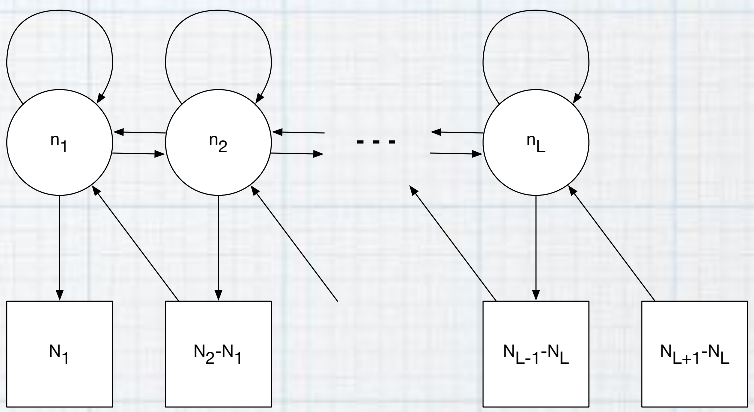

The index of QM

$$I = \frac{1}{|W|}\text{JK-Res}_{\vec{\eta}=\zeta\vec{1}}\left[g(u)d^r u\right]$$

$$g(u) = \left(\frac{1}{2\sinh\frac{-2\mu+\gamma}{2}}\right)^{\sum_{l=1}^{L} n_l} \prod_{l=1}^{L}\left(\prod_{i\neq j}^{n_l}\frac{\sinh\frac{u_i^{(l)}-u_j^{(l)}}{2}}{\sinh\frac{u_i^{(l)}-u_j^{(l)}-2\mu+\gamma}{2}}\right)\left(\prod_{i,j=1}^{n_l}\frac{\sinh\frac{u_i^{(l)}-u_j^{(l)}-2\mu-\gamma}{2}}{\sinh\frac{u_i^{(l)}-u_j^{(l)}-2\gamma}{2}}\right)$$

$$\times\left(\prod_{i=1}^{n_{l+1}}\prod_{j=1}^{n_l}\frac{\sinh\frac{u_i^{(l+1)}-u_j^{(l)}-2\mu+\gamma}{2}}{\sinh\frac{u_i^{(l+1)}-u_j^{(l)}}{2}}\right)\left(\prod_{i=1}^{n_{l+1}}\prod_{j=1}^{n_l}\frac{\sinh\frac{-u_i^{(l+1)}+u_j^{(l)}+2\gamma}{2}}{\sinh\frac{-u_i^{(l+1)}+u_j^{(l)}+2\mu+\gamma}{2}}\right)$$

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$$I = \prod_{l=1}^{L}\left(\prod_{a,b=1}^{N_l}\prod_{k=1}^{n_a^{(l)}-n_b^{(l)}}\frac{\sinh\frac{m_a-m_b-2\mu+2(k-\frac{1}{2})\gamma}{2}}{\sinh\frac{m_a-m_b+2(k-1)\gamma}{2}}\right)\left(\prod_{a=1}^{N_{l+1}}\prod_{b=1}^{N_l}\prod_{k=1}^{n_b^{(l)}-n_a^{(l+1)}}\frac{\sinh\frac{m_a-m_b-2\mu-2(k-\frac{1}{2})\gamma}{2}}{\sinh\frac{m_a-m_b-2k\gamma}{2}}\right)$$

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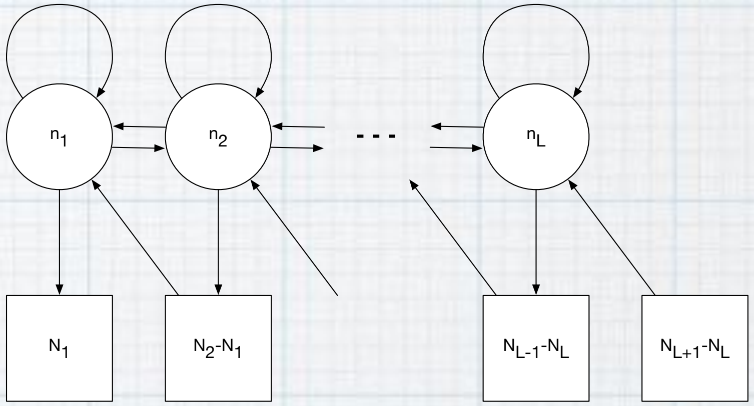

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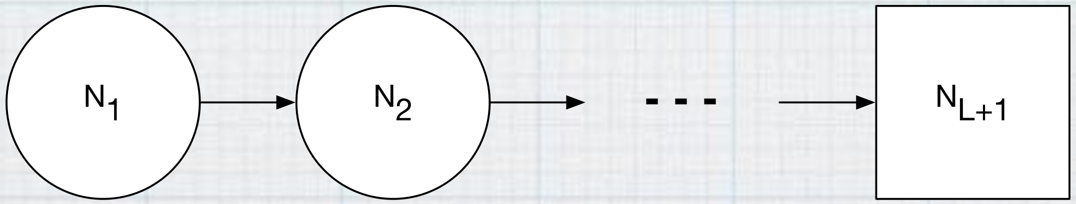
reproduces the known vortex partition function of $T_\rho[SU(N)]$

Bullimore-H.Kim-Koroteev '14

$$I = \prod_{l=1}^{L}\left(\prod_{a,b=1}^{N_l}\prod_{k=1}^{n_a^{(l)}-n_b^{(l)}}\frac{\sinh\frac{m_a-m_b-2\mu+2(k-\frac{1}{2})\gamma}{2}}{\sinh\frac{m_a-m_b+2(k-1)\gamma}{2}}\right)\left(\prod_{a=1}^{N_{l+1}}\prod_{b=1}^{N_l}\prod_{k=1}^{n_b^{(l)}-n_a^{(l+1)}}\frac{\sinh\frac{m_a-m_b-2\mu-2(k-\frac{1}{2})\gamma}{2}}{\sinh\frac{m_a-m_b-2k\gamma}{2}}\right)$$

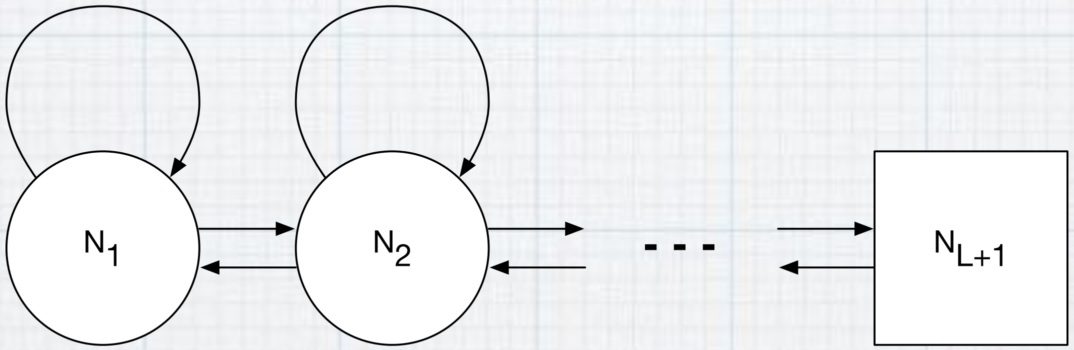
Example: N=2 linear quiver gauge theories

* Extensions to N=2 theories and vortices therein
* Branes setups are not clear.
* The N=2 deformation of $T_\rho[SU(N)]$
* Real mass for $U(1)_A \subset SU(2)^2$ R-symmetry



Example: N=2 linear quiver gauge theories

The N=2 deformation accompanies various CS/BF terms.



The bifundamentals integrated out

$$k^{(l)} = \frac{N_{l-1} + N_{l+1}}{2},$$

$$k^{(l,l+1)}_{U(1)} = -\frac{1}{2}$$

The adjoints integrated out

$$k^{(l)}_{SU(N)} = -N_l$$

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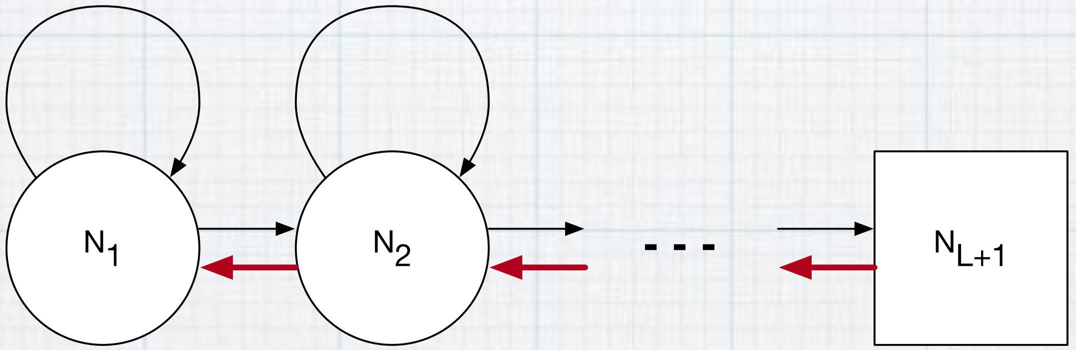

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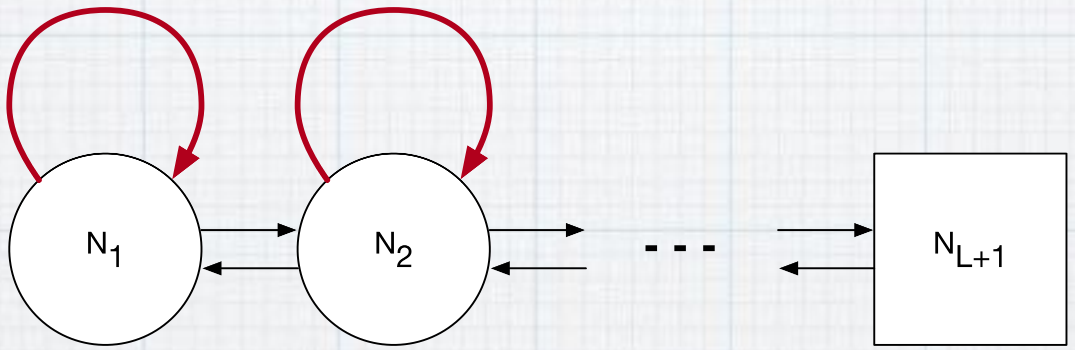

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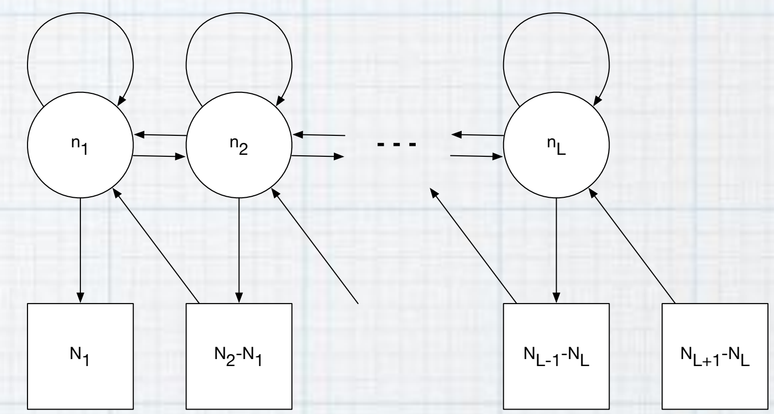
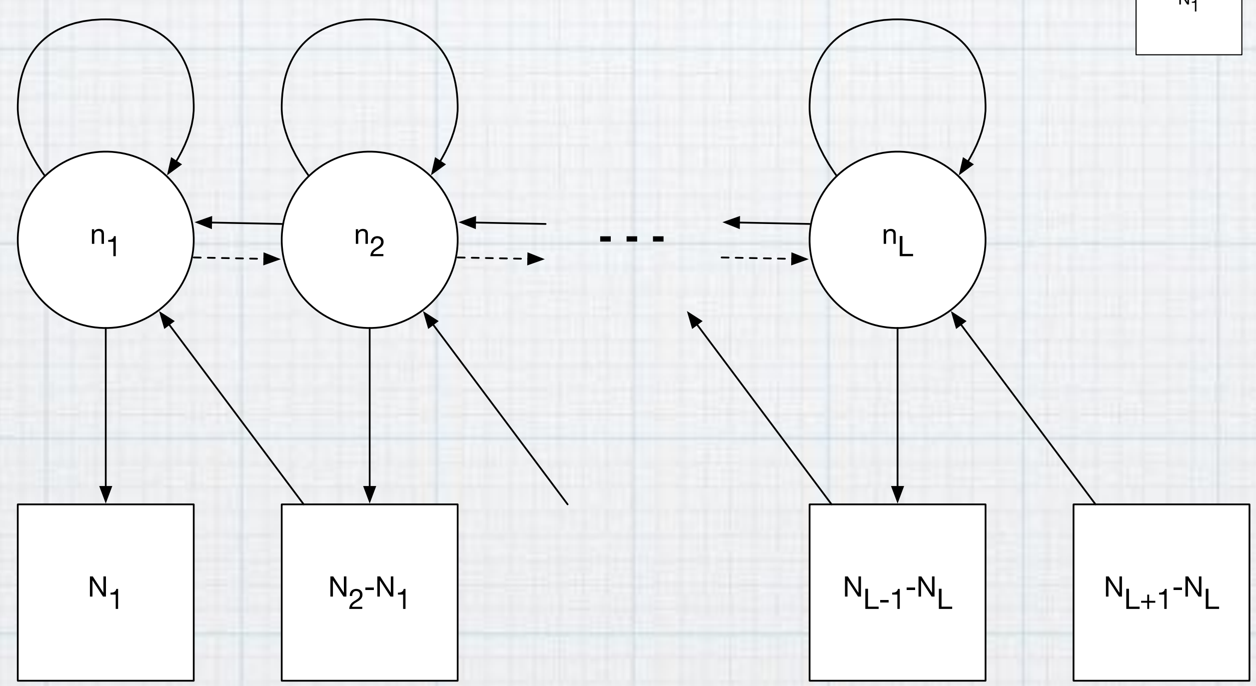


The deformation of vortex QM



The multiplets integrated out induce various gauge/flavor Wilson lines.

Example: N=2 linear quiver gauge theories

The 3d CS/BF interactions and their QM counterparts

Y.Kim-K.Lee '92, Collie-Tong '08, Collie '08

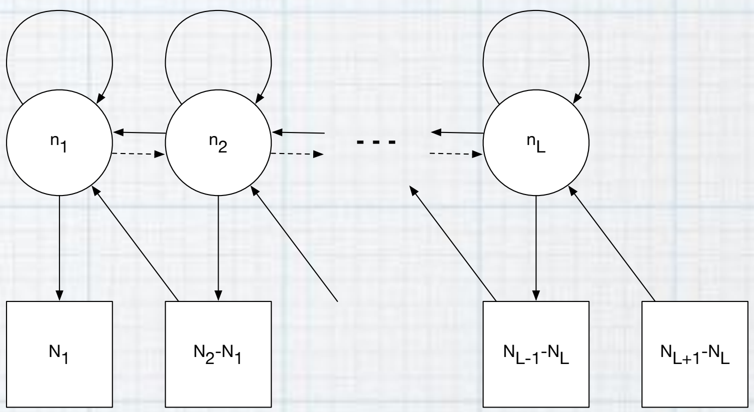
U(N) CS → $$\prod_{l=1}^{L} e^{k^{(l)}\sum_{i=1}^{n_l} u_i^{(l)}},$$

shifted U(1) CS → $$\prod_{l=1}^{L} e^{\Delta k_{U(1)}^{(l)}\left(n_l^2\gamma + n_l\sum_{k=1}^{l}\sum_{a=1}^{N'_k} m_a^{(k)}\right)},$$

BF → $$\prod_{l=1}^{L-1} e^{k_{U(1)}^{(l,l+1)}\left(2n_l n_{l+1}\gamma + n_l\sum_{k=1}^{l+1}\sum_{a=1}^{N'_k} m_a^{(k)} + n_{l+1}\sum_{k=1}^{l}\sum_{a=1}^{N'_k} m_a^{(k)}\right)}$$

Those will be important in the later discussions about Seiberg-like dualities for N=2 linear quiver theories.

Example: N=2 linear quiver gauge theories



The QM index

$$I = \frac{1}{|W|}\text{JK-Res}_{\vec{\eta}=\zeta\vec{1}}\left[g(u)d^r u\right]$$

$$g(u) = W(u)\prod_{l=1}^{L}\frac{\left(\prod_{i\neq j}^{n_l}\sinh\frac{u_i^{(l)}-u_j^{(l)}}{2}\right)\left(\prod_{i=1}^{n_{l+1}}\prod_{j=1}^{n_l}\sinh\frac{u_i^{(l+1)}-u_j^{(l)}-2\gamma}{2}\right)}{\left(\prod_{i,j=1}^{n_l}\sinh\frac{u_i^{(l)}-u_j^{(l)}-2\gamma}{2}\right)\left(\prod_{i=1}^{n_{l+1}}\prod_{j=1}^{n_l}\sinh\frac{u_i^{(l+1)}-u_j^{(l)}}{2}\right)}$$

$$\times\frac{1}{\left(\prod_{i=1}^{n_l}\prod_{b=N_{l-1}+1}^{N_l}\sinh\frac{u_i^{(l)}-m_b-\gamma}{2}\right)\left(\prod_{j=1}^{n_l}\prod_{a=N_l+1}^{N_{l+1}}\sinh\frac{-u_j^{(l)}+m_a-\gamma}{2}\right)}$$

$$I = W\prod_{l=1}^{L}\left(\prod_{a\neq b}^{N_l}\prod_{k=1}^{n_a^{(l)}-n_b^{(l)}}\sinh\frac{m_a-m_b+2(k-1)\gamma}{2}\right)^{-1}\left(\prod_{a=1}^{N_{l+1}}\prod_{b=1}^{N_l}\prod_{k=1}^{n_b^{(l)}-n_a^{(l+1)}}\sinh\frac{m_a-m_b-2k\gamma}{2}\right)^{-1}$$

Back to Seiberg-like dualities in 3d

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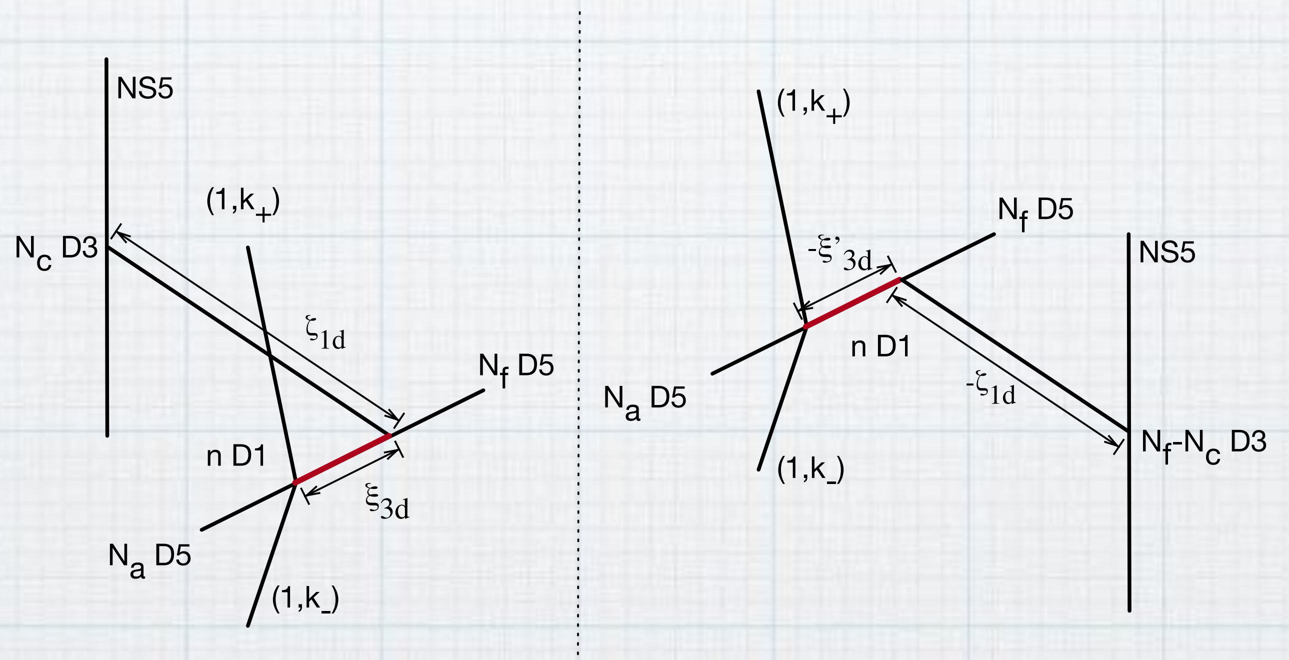
Recall the brane realization of the Aharony duality.

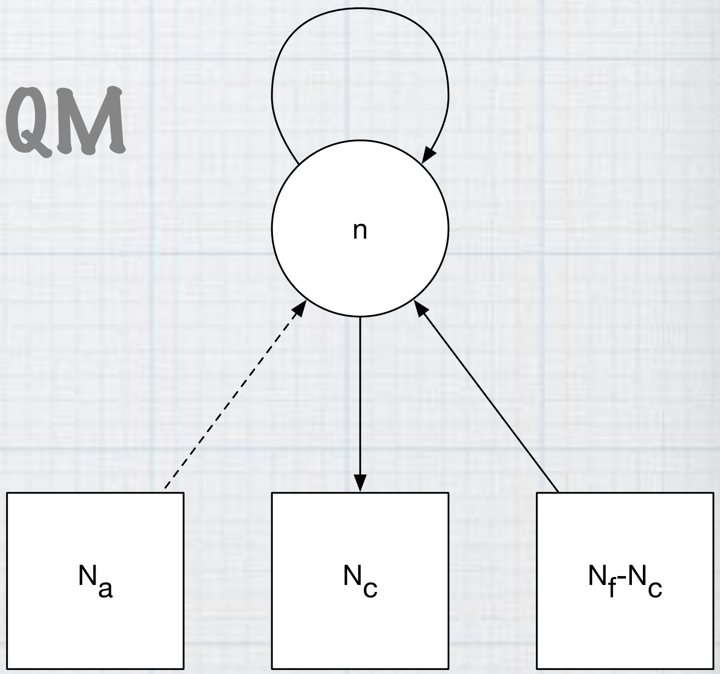


The presence of vortices is realized as the insertion of D1s.

What is the QM interpretation of this brane motion?

Aharony duality

* The wall-crossing controlled by 1d FI
* An Aharony dual pair share the same vortex QM; the only difference is 1d FI.
* The Aharony duality = the wall-crossing of vortex QM
* The QM index



$$I^n = \frac{1}{|W|}\text{JK-Res}_{\vec{\eta}=\zeta\vec{1}}\left[g^n(u)d^n u\right]$$

$$g^n(u) = \frac{e^{\kappa\sum_{i=1}^n u_i}\left(\prod_{i\neq j}^n \sinh\frac{u_i-u_j}{2}\right)\left(\prod_{j=1}^n\prod_{a=1}^{N_a}\sinh\frac{u_j-\tilde{m}_a+\mu-\gamma}{2}\right)}{\left(\prod_{i,j=1}^n \sinh\frac{u_i-u_j-2\gamma}{2}\right)\left(\prod_{i=1}^n\prod_{b=1}^{N_c}\sinh\frac{u_i-m_b-\mu-\gamma}{2}\right)\left(\prod_{j=1}^n\prod_{a=N_c+1}^{N_f}\sinh\frac{-u_j+m_a+\mu-\gamma}{2}\right)}$$

Aharony duality

The QM indices for different FI parameters

The choice of FI determines the poles contributing to the JK-residue.

E.g., the 1-vortex indices:

$$I_{\zeta>0} = \text{JK-Res}_{\eta=\zeta}\left[g(u)du\right] = \sum_{Q(u*)>0} \text{Res}_{u=u*}\left[g(u)du\right]$$

-> the vortex partition function of the original 3d theory CH-H.Kim-J.Park '12

$$I_{\zeta<0} = \text{JK-Res}_{\eta=\zeta}\left[g(u)du\right] = -\sum_{Q(u*)<0} \text{Res}_{u=u*}\left[g(u)du\right]$$

-> the vortex partition function of the dual 3d theory CH-H.Kim-J.Park '12

Aharony duality

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$$= I_{\zeta<0}$$

Aharony duality

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The choice of FI determines the poles contributing to the JK-residue.

E.g., the 1-vortex indices:

$$I_{\zeta>0} = \text{JK-Res}_{\eta=\zeta}\left[g(u)du\right] = \sum_{Q(u*)>0} \text{Res}_{u=u*}\left[g(u)du\right]$$

$$\begin{aligned} I_{\zeta>0} &= \text{JK-Res}_{\eta=-\zeta}\left[g(u)du\right] \\ &= -\sum_{Q(u*)<0} \text{Res}_{u=u*}\left[g(u)du\right] \boxed{-\text{Res}_{u=\pm\infty}\left[g(u)du\right]} \end{aligned}$$

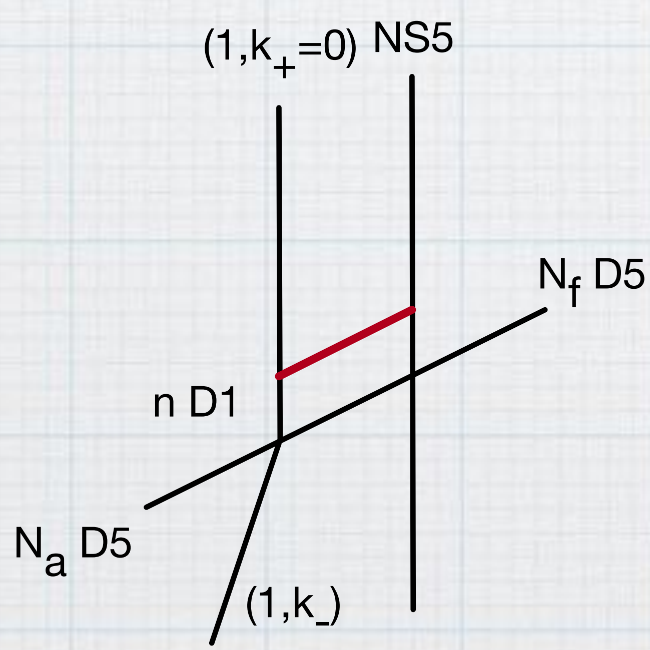
a nontrivial wall-crossing at ζ=0

Aharony duality

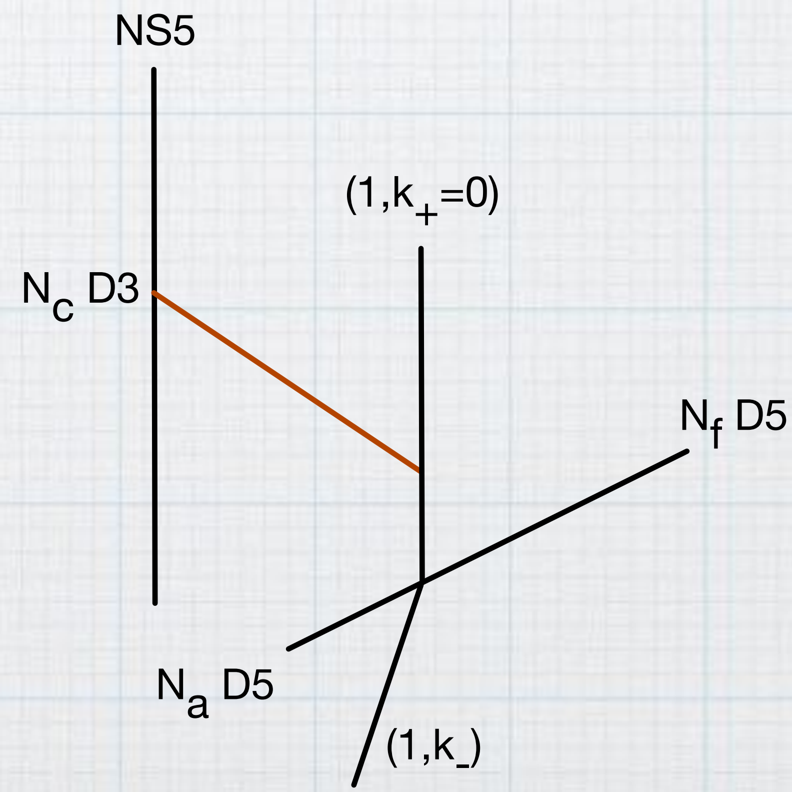
The necessary condition for the nontrivial wall-crossing

$$g^n(u) \sim e^{\left(\kappa \pm \frac{N_a - N_f}{2}\right) u_i} \quad \Rightarrow \quad \pm\kappa - \frac{N_f - N_a}{2} = 0$$

The branes at ζ=0



C.f., the branes at ξ=0



Aharony duality

We evaluate the exact wall-crossing factor:

$$\sum_{n=0}^{\infty} w^n I^n_{\zeta>0} = \left(\sum_{n=0}^{\infty} w^n I^n_{\zeta<0}\right)$$
$$\times \, \mathrm{PE}\left[\frac{\delta_{2\kappa, N_a - N_f}\tau^{-\frac{N_f+N_a}{2}} x^{-N_c+\frac{N_f+N_a}{2}+1} - \delta_{2\kappa, N_f-N_a}\tau^{\frac{N_f+N_a}{2}} x^{N_c-\frac{N_f+N_a}{2}+1}}{1-x^2}\mathbf{w}\right]$$

Aharony duality

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the BPS index of V_+ & V_-, which describe the Coulomb moduli space

Aharony duality

We evaluate the exact wall-crossing factor:

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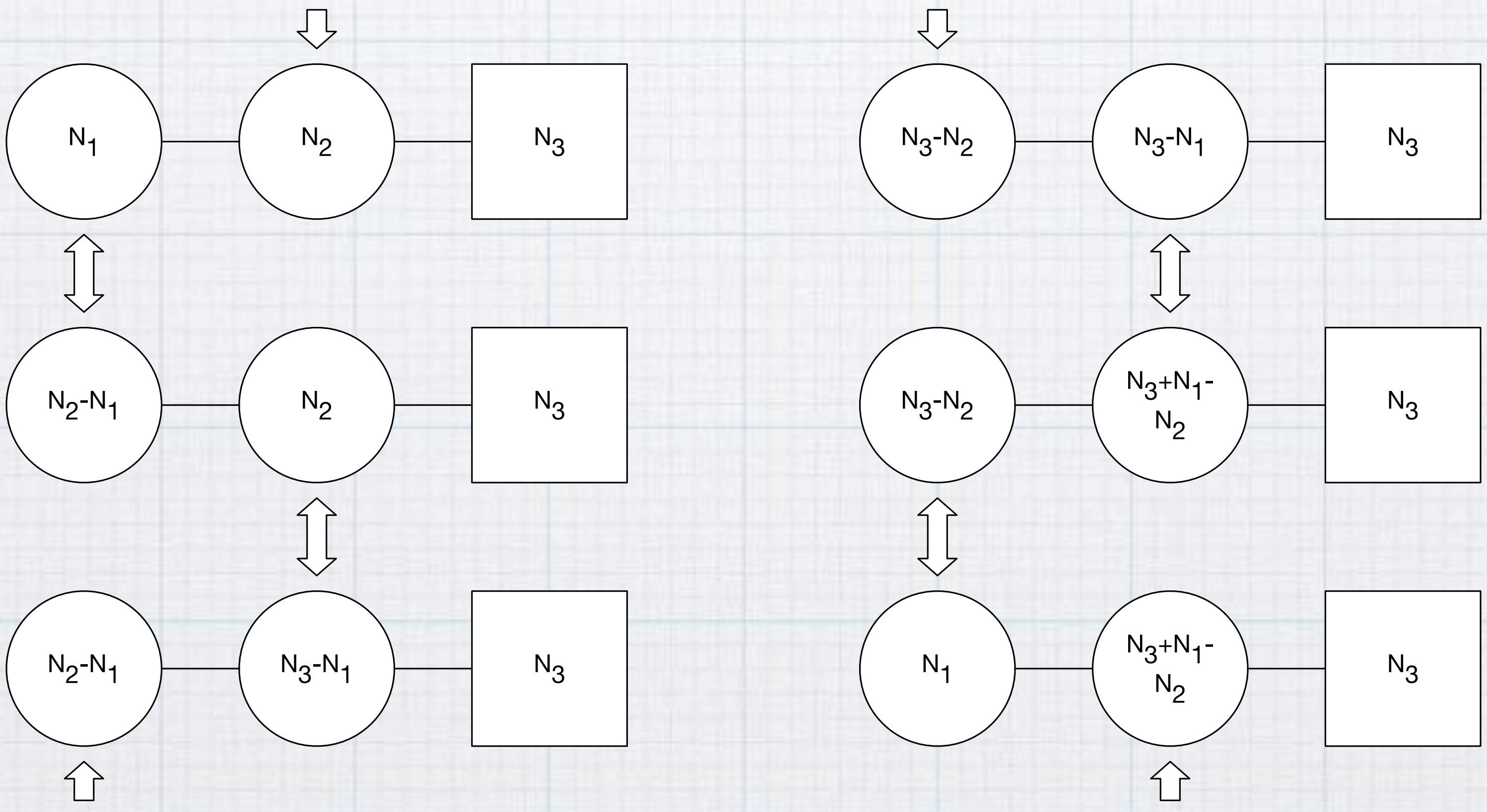
* The Aharony duality corresponds to the wall-crossing of vortex QM.
* The escaping states at the wall in QM are related to the Coulomb branch operators in the 3d theory.
* The same interpretation holds for the Seiberg-like duality of the 3d N=4 QCD. H.Kim-J.Kim-S.Kim-K.Lee '12, Yaakov '13, Gaiotto-Koroteev '13

Linear quiver: $T_\rho[SU(N)]$

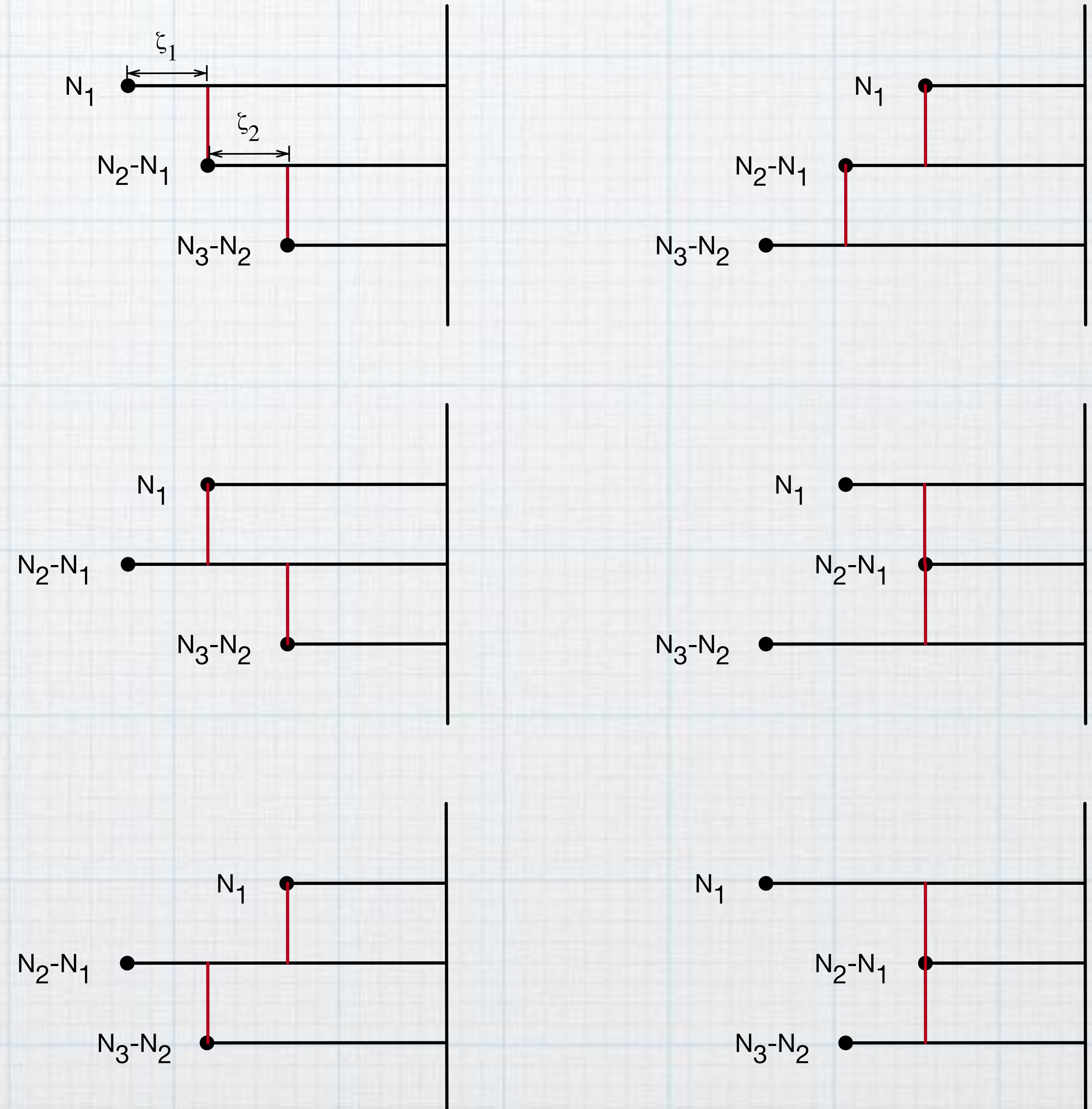
The Seiberg-like dual chain for a three-node theory



Dual theories contain additional free twisted hypers, which describe Coulomb branches of the moduli space.

Linear quiver: $T_\rho[SU(N)]$

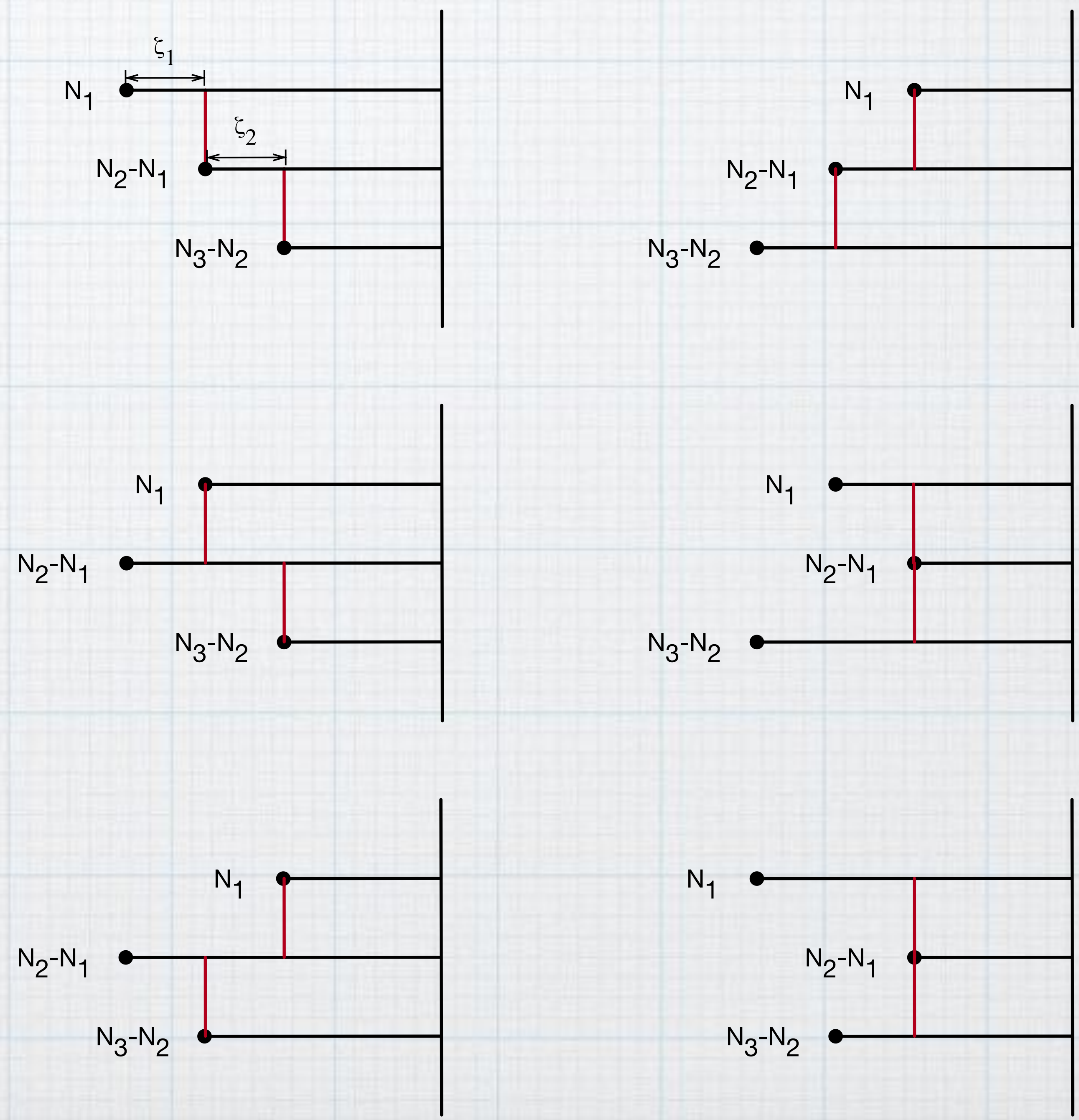
The brane realization of the Seiberg-like dualities of $T_\rho[SU(N)]$



This brane motion corresponds to the wall-crossing of D1 QM

Linear quiver: $T_\rho[SU(N)]$

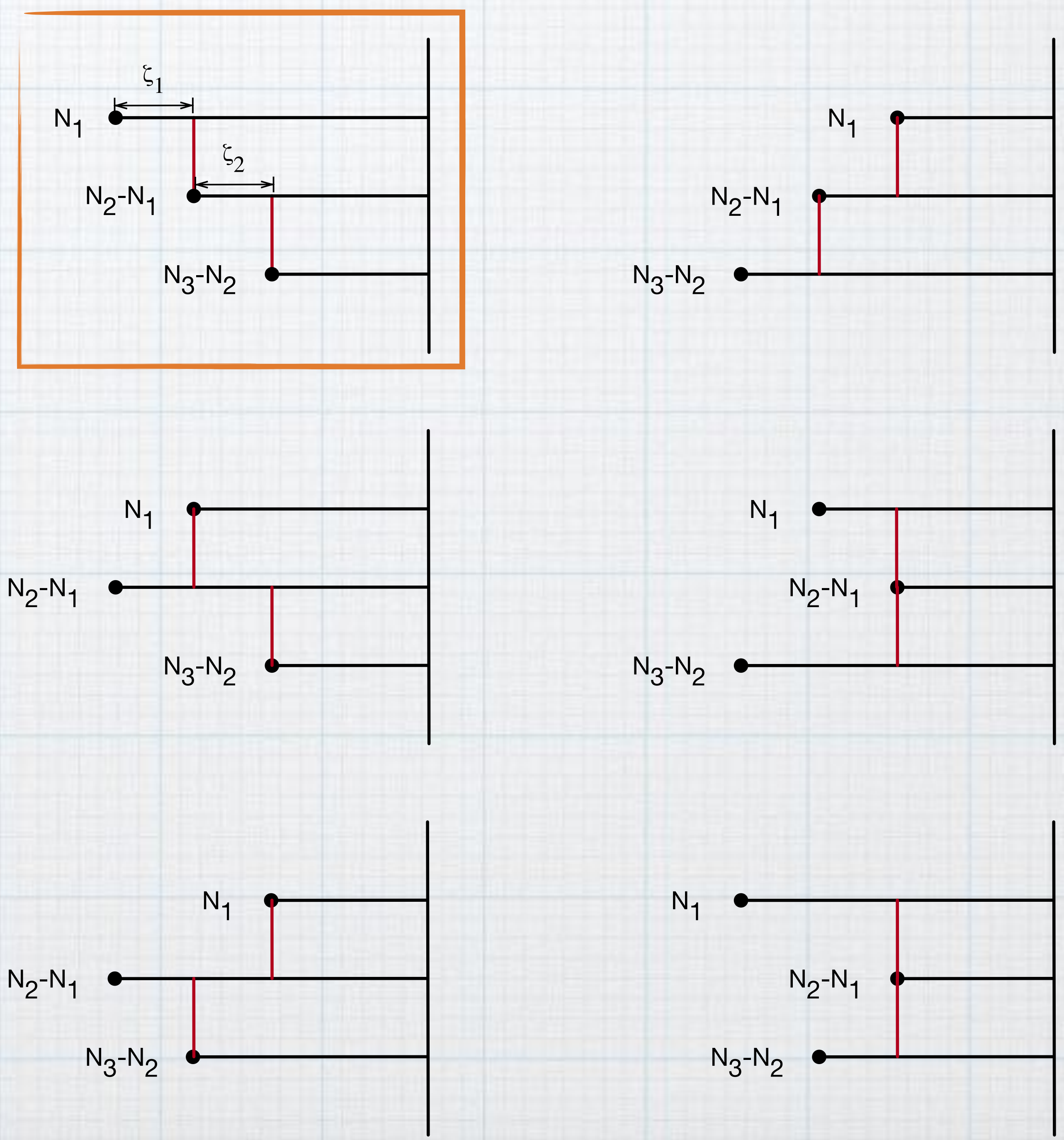
The brane realization of the **Seiberg-like dualities** of $T_\rho[SU(N)]$



This brane motion corresponds to the **wall-crossing** of D1 QM

Linear quiver: $T_\rho[SU(N)]$

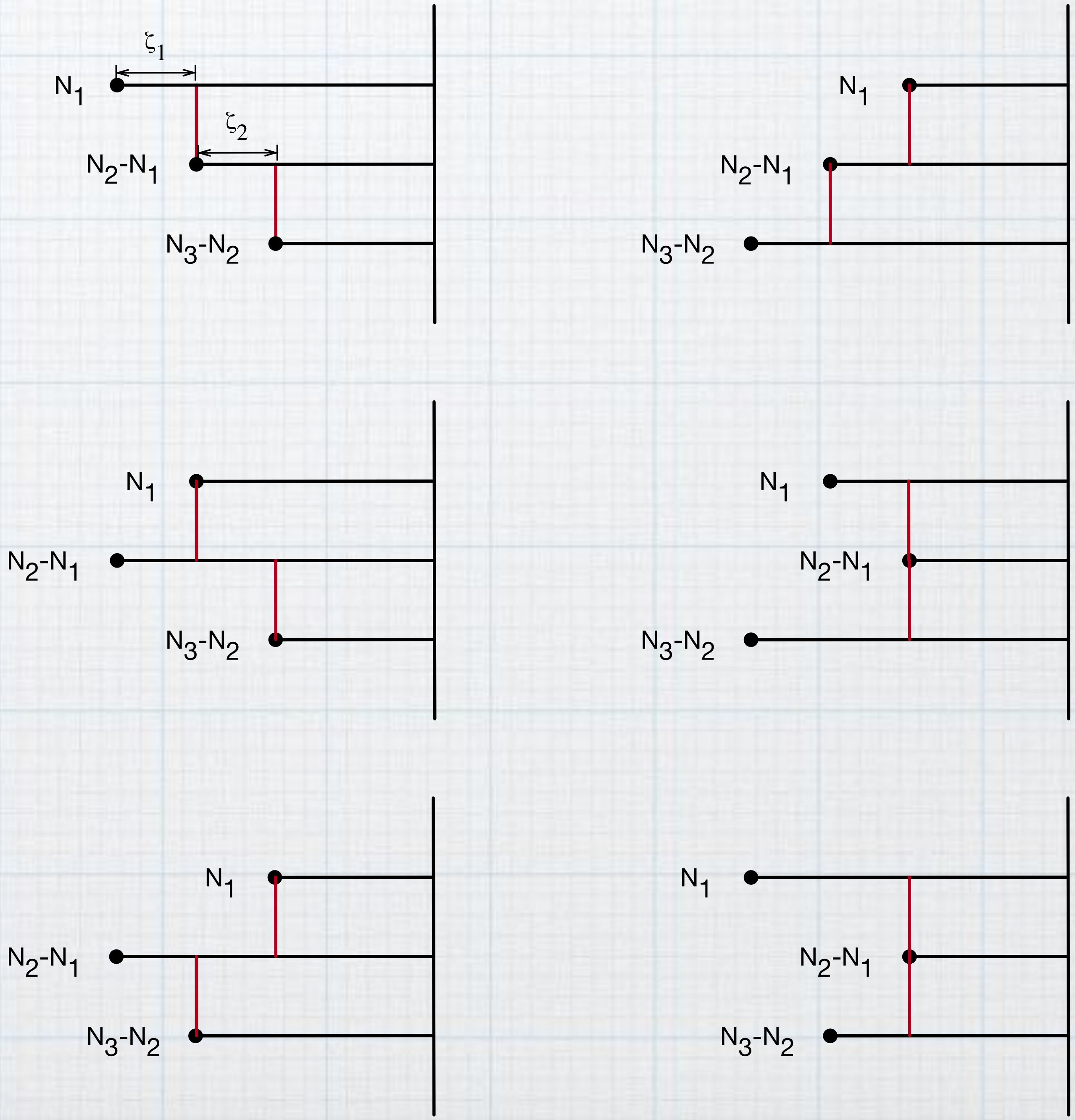
The vortex partition functions are only known for the first and fourth theories, which have all positive or all negative 3d FI parameters.



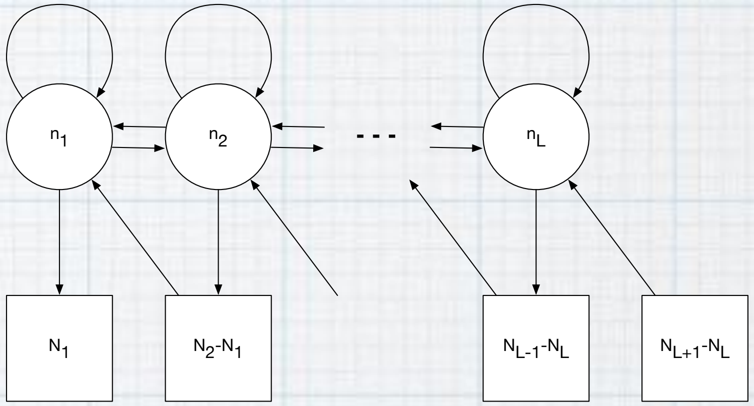
Linear quiver: $T_\rho[SU(N)]$

The vortex partition functions are only known for the first and fourth theories, which have all positive or all negative 3d FI parameters.

The other theories have both positive and negative FI parameters, the factorization cannot compute the vortex partition function for such cases.



Linear quiver: $T_\rho[SU(N)]$

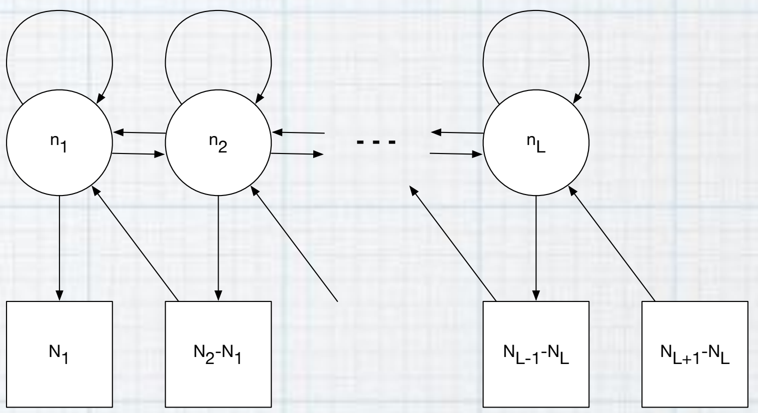


The wall-crossing factors:

$$
\begin{aligned}
Z_1/Z_2 &= Z^{\text{wall}}_{N_1,N_2}(w_1),\\
Z_2/Z_3 &= Z^{\text{wall}}_{N_2,N_3+N_2-N_1}(w_1 w_2),\\
Z_3/Z_4 &= Z^{\text{wall}}_{N_2-N_1,N_3-N_1}(w_2),\\
Z_4/Z_5 &= Z^{\text{wall}}_{N_3-N_1,2N_3-N_2}(w_1),\\
Z_5/Z_6 &= Z^{\text{wall}}_{N_3-N_2,N_3-N_2+N_1}(w_1 w_2),\\
Z_6/Z_1 &= Z^{\text{wall}}_{N_3-N_2+N_1,N_1+N_3}(w_2)
\end{aligned}
$$

$$
Z^{\text{wall}}_{N,M}(w) = \text{PE}\left[\frac{\sinh\frac{(2N-M)(-2\mu+\gamma)}{2}\sinh\frac{-2\mu-\gamma}{2}}{\sinh\frac{-2\mu+\gamma}{2}\sinh\frac{-2\gamma}{2}}w\right]
$$

Linear quiver: $T_\rho[SU(N)]$



The wall-crossing factors:

$$Z_1/Z_2 = Z^{\text{wall}}_{N_1,N_2}(w_1),$$
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$$Z_3/Z_4 = Z^{\text{wall}}_{N_2-N_1,N_3-N_1}(w_2),$$
$$Z_4/Z_5 = Z^{\text{wall}}_{N_3-N_1,2N_3-N_2}(w_1),$$
$$Z_5/Z_6 = Z^{\text{wall}}_{N_3-N_2,N_3-N_2+N_1}(w_1 w_2),$$
$$Z_6/Z_1 = Z^{\text{wall}}_{N_3-N_2+N_1,N_1+N_3}(w_2)$$

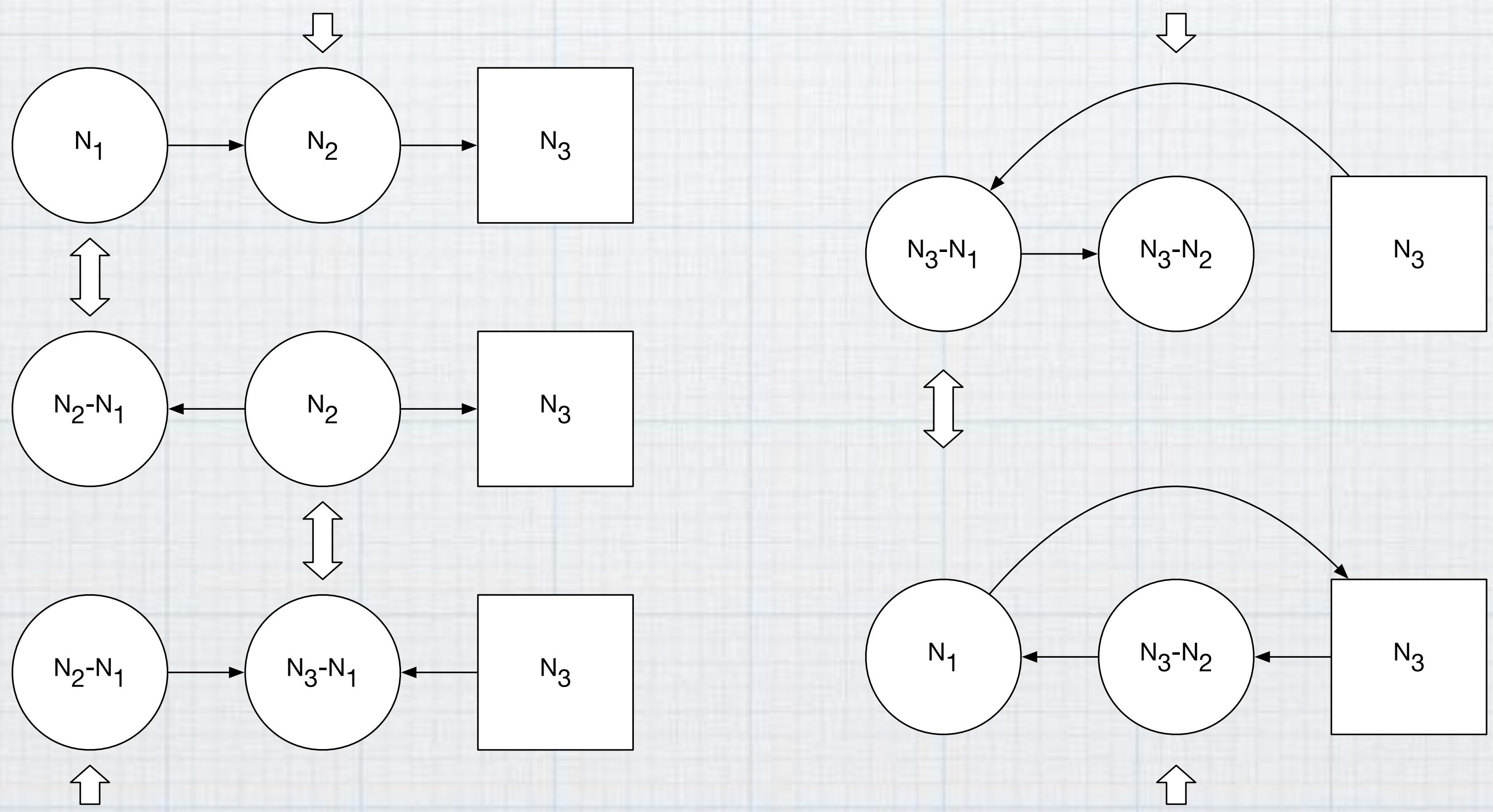
$$Z^{\text{wall}}_{N,M}(w) = \mathrm{PE}\left[\frac{\sinh\frac{(2N-M)(-2\mu+\gamma)}{2}\sinh\frac{-2\mu-\gamma}{2}}{\sinh\frac{-2\mu+\gamma}{2}\sinh\frac{-2\gamma}{2}}w\right]$$

<-> the free twisted hypers describing Coulomb branches of the moduli space

Linear quiver: N=2 theories

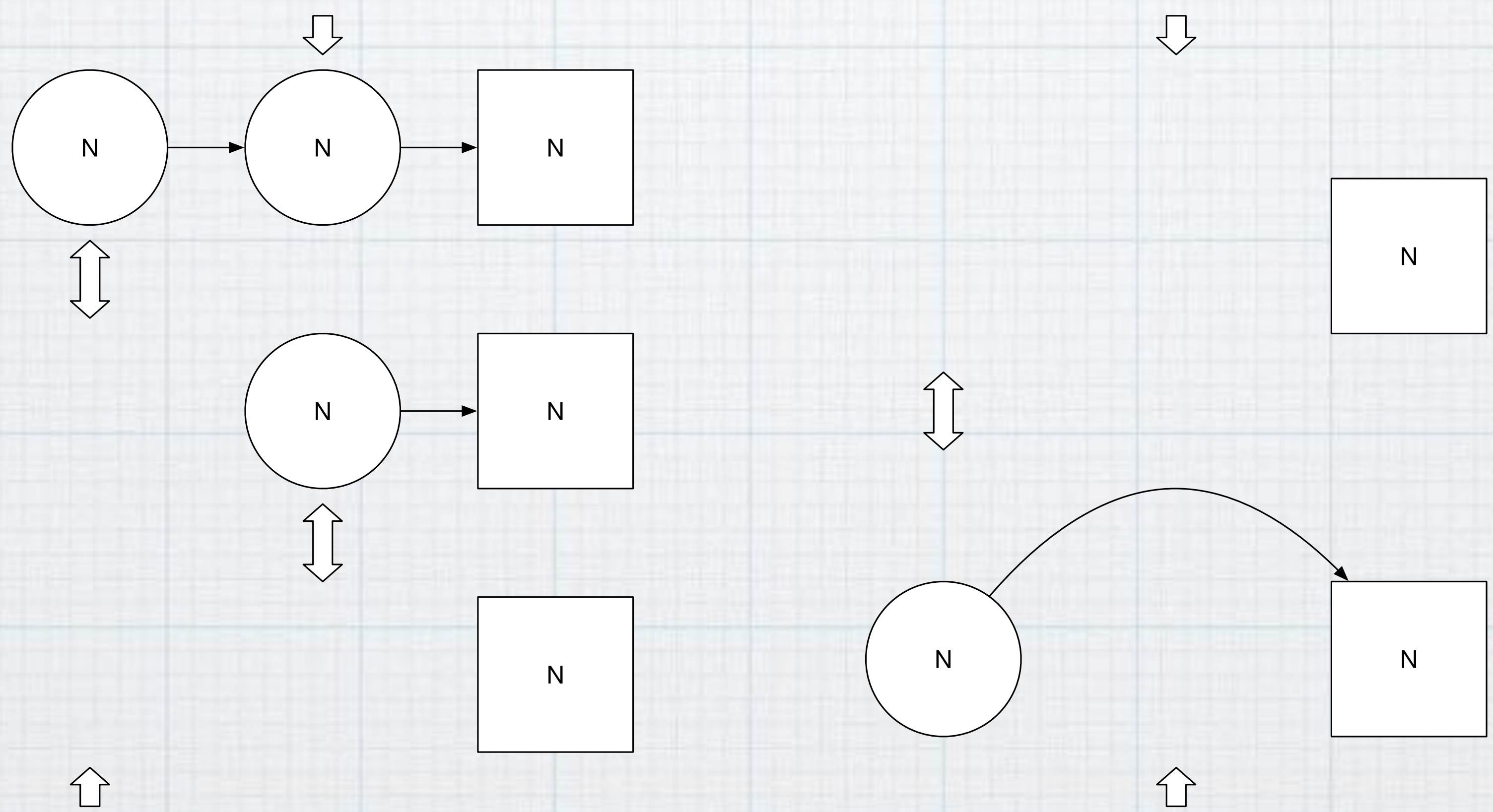
Brane setups are not clear. But still...

A typical Seiberg-dual chain of a N=2 linear quiver theory



Linear quiver: N=2 theories

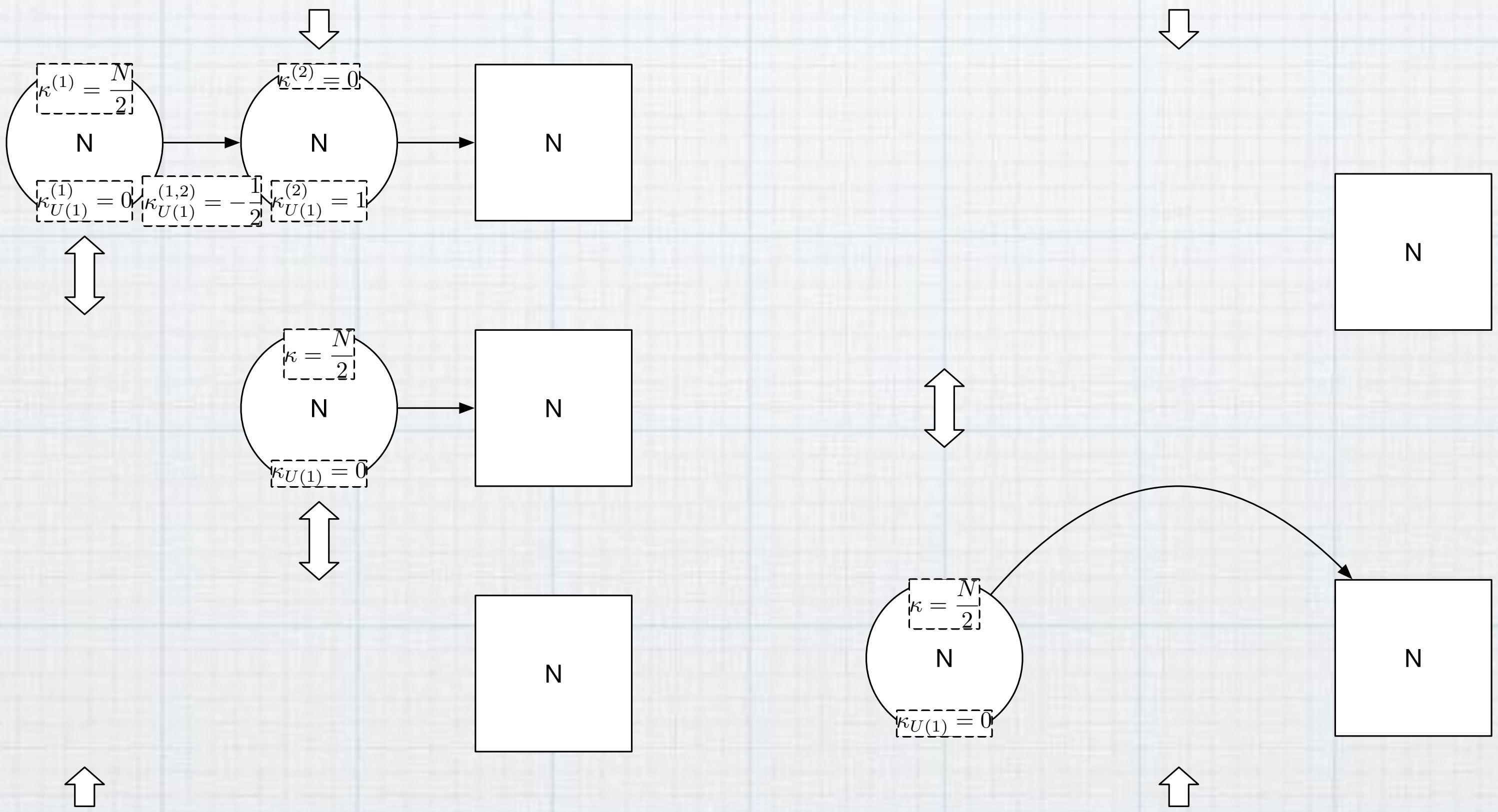
Let's consider an (N,N,N)-type example.



Dual theories contain additional gauge singlet chirals, which describe the Coulomb branches of the moduli space.

Linear quiver: N=2 theories

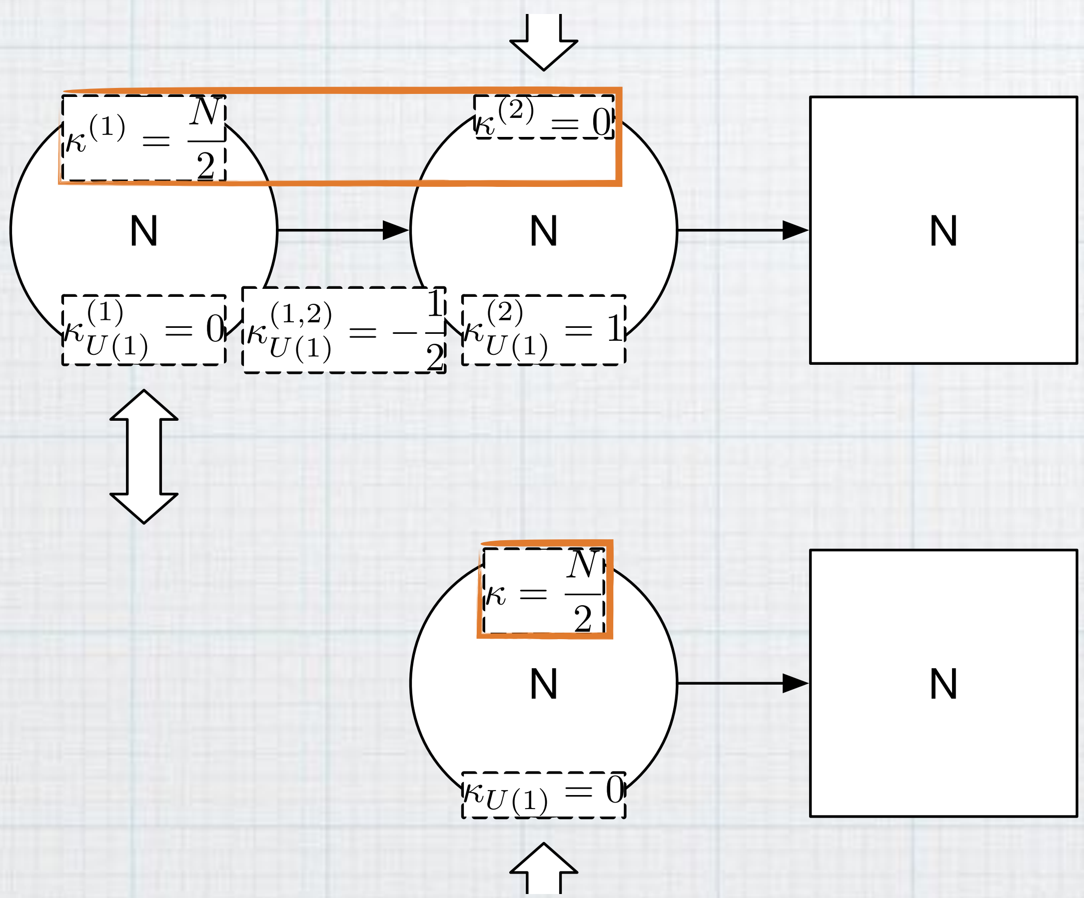
Let's consider an (N,N,N)-type example.



Dual theories contain additional gauge singlet chirals, which describe the Coulomb branches of the moduli space.

Linear quiver: N=2 theories

CS levels under the duality



The U(N) CS levels under the duality:

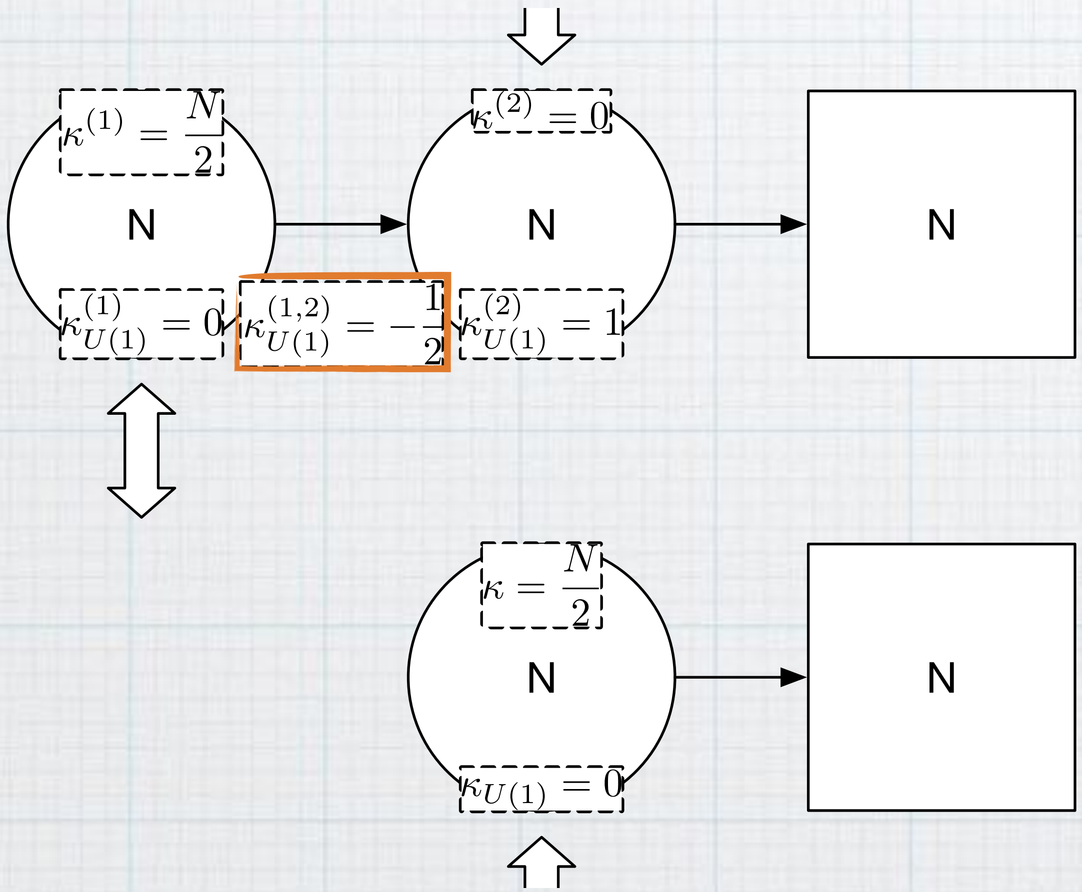
Benini-Closset-Cremonesi '11

$\kappa^{(1)} \to -\kappa^{(1)}$,

$\kappa^{(2)} \to \kappa^{(2)} + \kappa^{(1)}$

Linear quiver: N=2 theories

CS levels under the duality

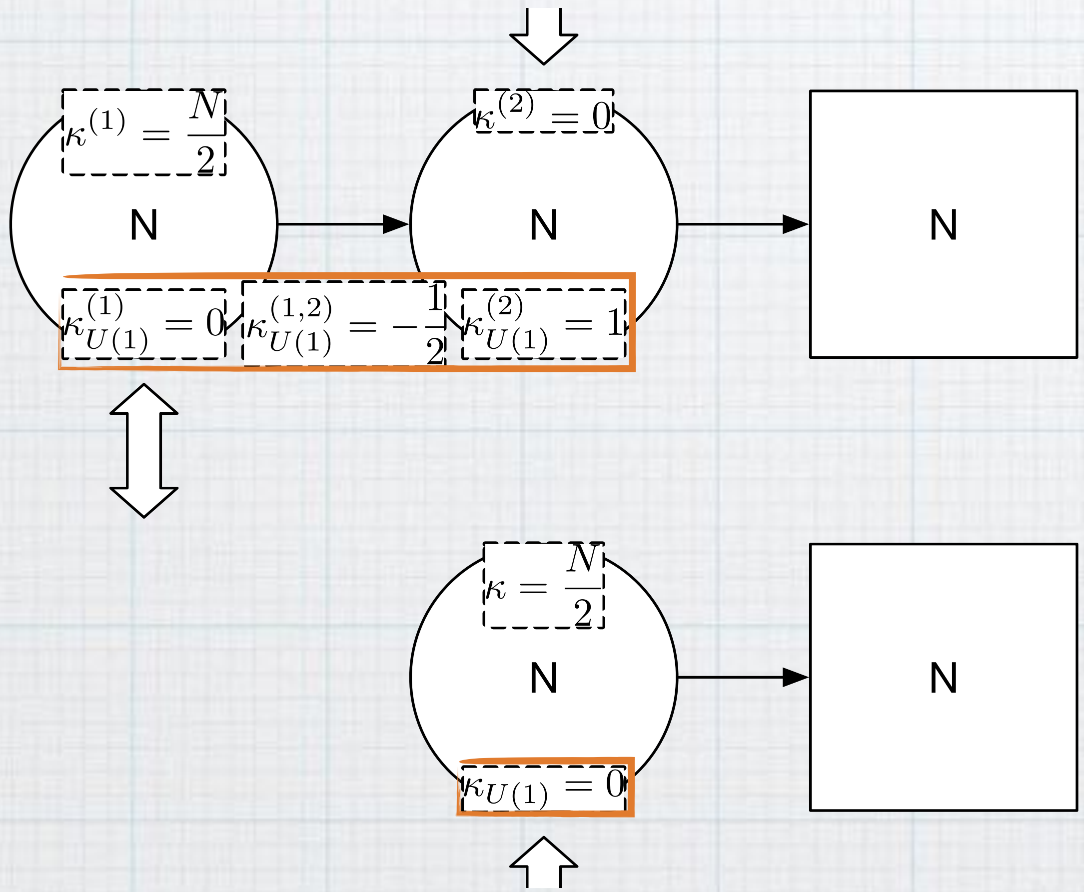


There must be the level half BF interaction between the gauge nodes.

What is its effect on the duality?

Linear quiver: N=2 theories

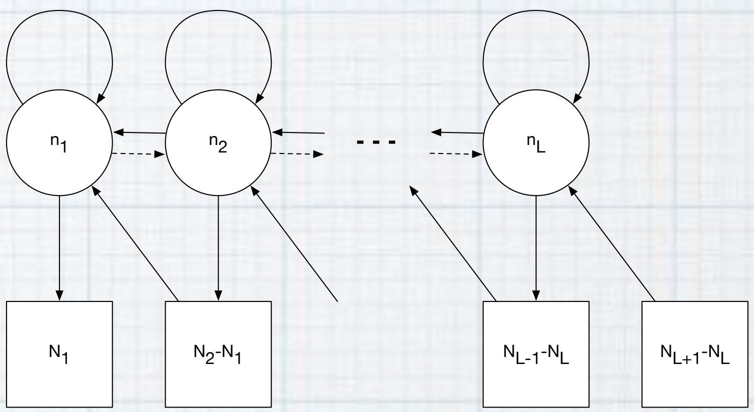
CS levels under the duality



The shift of a U(1) CS level:

$\kappa^{(2)}_{U(1)} \to \kappa^{(2)}_{U(1)} - 1$

Linear quiver: N=2 theories



The QM indices for different FIs

$$Z_{\mathbf{1}} = \sum_{n_1}^{\infty}\sum_{n_2=0}^{\infty}(-w_1)^{n_1} w_2^{n_2} I_{\mathbf{1}}^{n_1,n_2} = \mathrm{PE}\left[-\frac{x^{\frac{N}{2}+1}w_1}{1-x^2} - \frac{x^{\frac{N}{2}+1}\tau^{\frac{N}{2}}w_1w_2}{1-x^2}\right],$$

$$Z_{\mathbf{2}} = \sum_{n_1}^{\infty}\sum_{n_2=0}^{\infty}(-w_1)^{n_1} w_2^{n_2} I_{\mathbf{2}}^{n_1,n_2} = \mathrm{PE}\left[-\frac{x^{\frac{N}{2}+1}\tau^{\frac{N}{2}}w_1w_2}{1-x^2}\right],$$

$$Z_{\mathbf{3}} = \sum_{n_1}^{\infty}\sum_{n_2=0}^{\infty}(-w_1)^{n_1} w_2^{n_2} I_{\mathbf{3}}^{n_1,n_2} = 1,$$

$$Z_{\mathbf{4}} = \sum_{n_1}^{\infty}\sum_{n_2=0}^{\infty}(-w_1)^{n_1} w_2^{n_2} I_{\mathbf{4}}^{n_1,n_2} = 1,$$

$$Z_{\mathbf{5}} = \sum_{n_1}^{\infty}\sum_{n_2=0}^{\infty}(-w_1)^{n_1} w_2^{n_2} I_{\mathbf{1}}^{n_1,n_2} = \mathrm{PE}\left[-\frac{x^{\frac{N}{2}+1}w_1}{1-x^2}\right]$$

-> reproduce the known vortex partition functions.

Linear quiver: N=2 theories

* The Seiberg-like dualities <-> the wall-crossings of vortex QM
* The wall-crossing factor <-> the gauge singlet chirals describing the Coulomb moduli space
* More examples such as a (1,1,N)-type theory.
* The U(1) CS/BF interactions are crucial to match the vortex partition functions under the dualities.
* General phenomenon for any 3d N=2 theory?

Conclusion

Conclusion

* We have constructed quantum mechanics descriptions of the 3d vortex moduli spaces.
* The Witten indices of those quantum mechanics gives the vortex partition functions. (C.f. factorizations of 3d partition functions)
* Seiberg-like duality -> wall-crossing of vortex quantum mechanics
* This relation implies a condition on asymptotic behavior of vortex quantum mechanics. (C.f. N=2 abelian linear quiver theories)
* A useful tool for investigating Seiberg-like dualities of linear quiver theories
* The vortex partition function equality <-> identities for 3d partition functions —relation to the Gauge/YBE correspondence for integrable systems?
* General phenomenon? Seiberg-like duality <- wall-crossing?